# Flaws in plate glass

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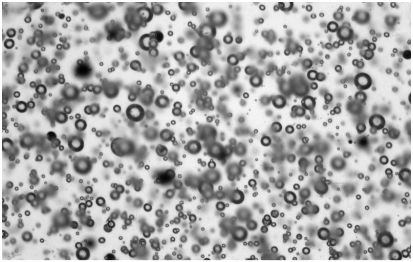
Mathematics in Industry Study Group, 2018

# Problem description

1. Flaws occur as bubbles in the glass and effect optical properties of the glass.
2. Bubbles are produced in the furnace (batch or chemical reactions).
3. Most bubbles rise to the surface and break.
4. If not removed in the furnace they remain in the plate glass.



# The aim

Our aim is to remove the few remaining small bubbles (0.03 $mm$), to ensure:

- Quality control and efficiency.
- Environmentally safe.

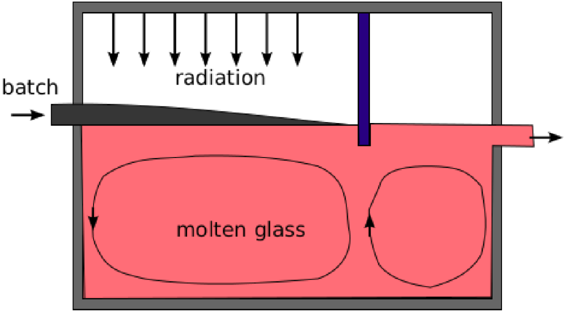
There are two procedures used to sweep up small bubbles:

1. Adding fining agents.
2. Introducing larger bubbles.

1. Air bubbles (Nitrogen and Oxygen) are carried into the furnace with the batch.
2. Chemical equation:

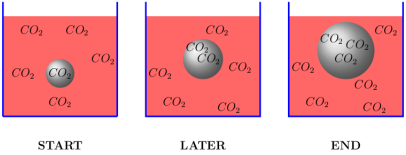
$$3SiO_2 + Na_2CO_3 \Leftrightarrow Na_2O + 3SiO_2 + CO_2$$

3. $CO_2$ and $N_2$ relatively insoluble $\therefore$ creates air bubble.
4. Assumption: only $CO_2$ in bubble.

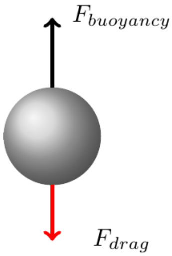


# Individual bubble behaviour

1. Bubble growth due to gas transfer across the surface of bubble.



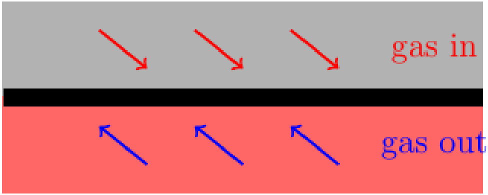

2. Bubble movement due to buoyancy.



1. Henry's law (Thermodynamic equilibrium):

$$c_l = Hp$$

$$\xrightarrow[\text{equilibrium}]{\text{THERMODYNAMIC}}$$

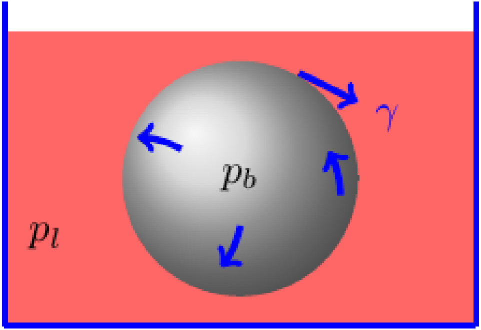



**Henry's law**

# Bubble growth

1. Laplace equation (Pressure of a bubble):

$$p_b = p_l + \frac{2\gamma}{R}$$




**Laplace's law**

# Bubble growth

1. Henry's law in a bubble:

$$c_b = H(p_l + \frac{2\gamma}{R})$$

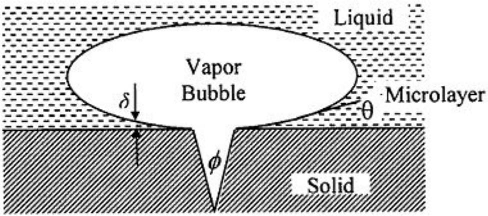
2. Critical radius:

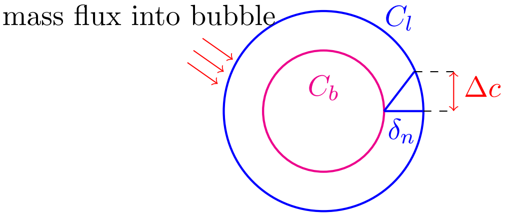
$$R_c = \frac{2\gamma}{\frac{c_l}{H} - p_l}$$

# Nucleation

Nucleation is the localized formation of a distinct thermodynamic phase. In a liquid it is the formation of gaseous bubbles.

1. **Homogeneous nucleation** Occurs spontaneously and randomly in the liquid (requires supersaturation).
2. **Heterogeneous nucleation** Occurs at nucleation site on surfaces contacting the liquid.



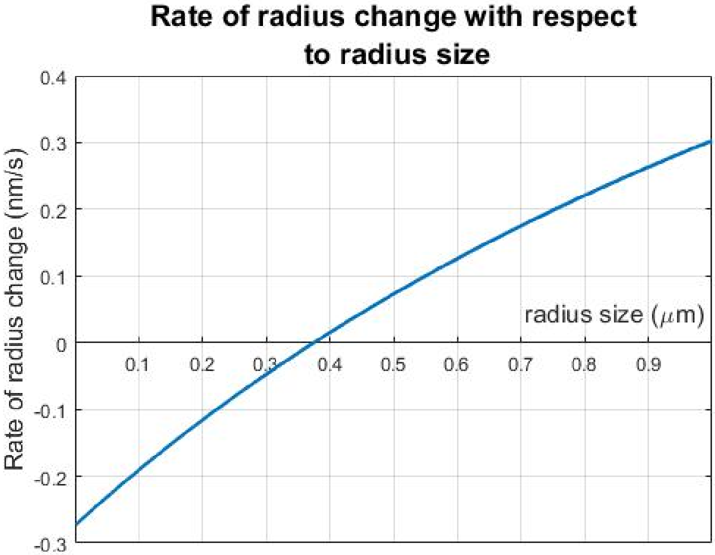



$$\frac{d}{dt}\left(\frac{4}{3}\pi R^3 \rho_{b(t)}\right) = k4\pi R^2 (C_l - C_{b(t)}) \tag{1}$$

$$\frac{dR}{dt} = \frac{\frac{k}{\rho_0}\left(c_l - H\left(p_l + \frac{2\gamma}{R}\right)\right)}{1 + \frac{2\epsilon}{3R}} \tag{2}$$

$$\frac{dR}{dt} = 0 \implies R_c = \frac{2\gamma}{\frac{c_l}{H} - p_l}$$

# Stability



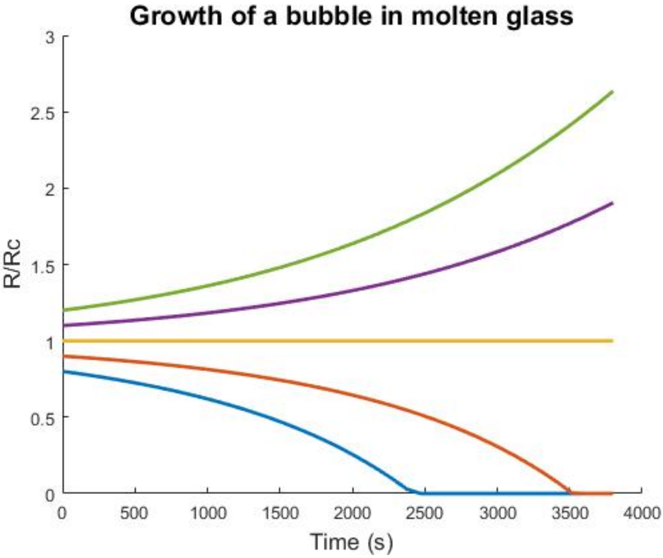
Rate of radius change with respect to radius size
Rate of radius change (nm/s)
radius size ($\mu$m)
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

- Henry's constant for $CO_2$ in $H_2O$ at room temperature:

$$K^0_{H,cp} = \frac{C_{aq}}{P_{gas}}.$$

- Conversion tool:

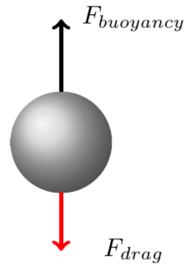
$$K_{H,cp} = K^0_{H,cp} \exp\left(C\left(\frac{1}{T} - \frac{1}{T^0}\right)\right)$$


Growth of a bubble in molten glass
R/Rc
3
2.5
2
1.5
1
0.5
0
0
500
1000
1500
2000
2500
3000
3500
4000
Time (s)

# Bubble movement

1. Force due to buoyancy:

$$F_{buoyancy} = \frac{4}{3}\pi R^3 \rho_s g$$




2. Stoke's drag force:

$$F_{drag} = 6\pi R \mu \dot{z}$$

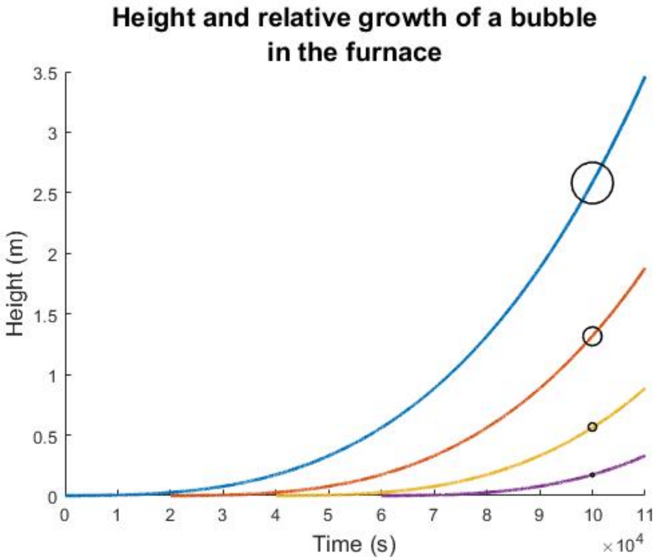
3. At terminal velocity forces are balanced:

$$\dot{z} = \frac{2}{9}\frac{R^2(t)\rho_s g}{\mu}$$

4. We can show that the growth of the bubble depends on the depth:

$$\frac{dR}{dt} = \frac{\frac{k}{\rho_0}\left(c_l - H\left(p_l(z) + \frac{2\gamma}{R}\right)\right)}{1 + \frac{2\epsilon}{3R}}$$

# Solutions

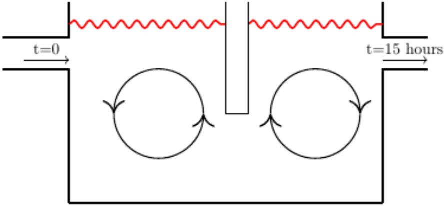


Height and relative growth of a bubble in the furnace
Height (m)
3.5
3
2.5
2
1.5
1
0.5
0
0
1
2
3
4
5
6
7
8
9
10
11
Time (s)
$\times 10^4$

# Observations and convection flow

1. $R_c$ at the top: $3.7 \times 10^{-7}\ m$
2. $R_c$ at the bottom: $4.2 \times 10^{-7}\ m$
3. Considering the convection flow
4. Convection flow speed:

$$v \sim 3 \times 10^{-4} m/s$$




5. Using:

$$\dot{z} = \frac{2}{9} \frac{R^2(t)\rho_s g}{\mu}$$

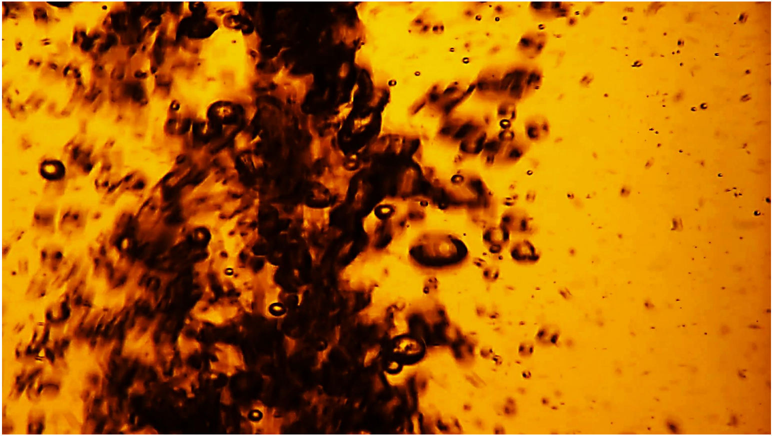
6. $3.7 \times 10^{-7} m < R < 2.5 \times 10^{-4} m.$

**Fining agents**

1. Fining agents are chemicals that remove bubbles from molten glass.
2. Arsenic oxide:
   - One of most efficient chemical fining agents
   - Highly toxic
   - Adding alkali and alkaline earth oxides makes it less volatile
3. Less noxious compounds in use:
   - Sodium chloride
   - Sodium sulphate
   - Sodium nitrate

**Introducing more bubbles**

1. Device that mechanically inserts bubbles into the molten glass
2. Experiment conducted by engineers



# Conclusion

1. Investigated how a bubble grows and moves in molten glass.
2. Critical range gives insight to problem bubbles.
3. Preliminary investigation to remove bubbles.
4. Future work
    - Two gas model
    - More accurate Henry's constant

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THANK YOU

FOR

YOUR

ATTENTION

ANY QUESTIONS?